香港中文大學
The Chinese University of Hong Kong

CSCI2510 Computer Organization

Lecture 02: Number and Character Representation

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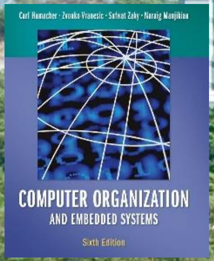
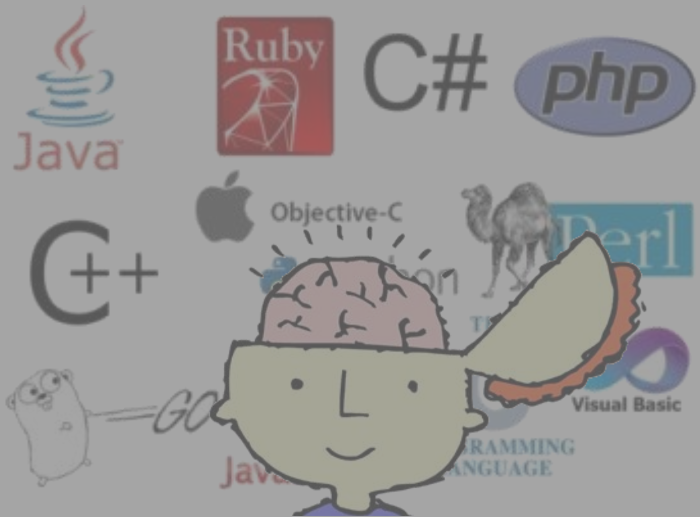


Reading: Chap. 1.4~1.5, 9.7~9.8

Recall: How to talk to the computer?



High-level Language

- Easy for programmer to understand
- Human understandable English words

Language Translation

Machine Language

- The computer's own language
- Binary numbers (All 1s and 0s)

Outline

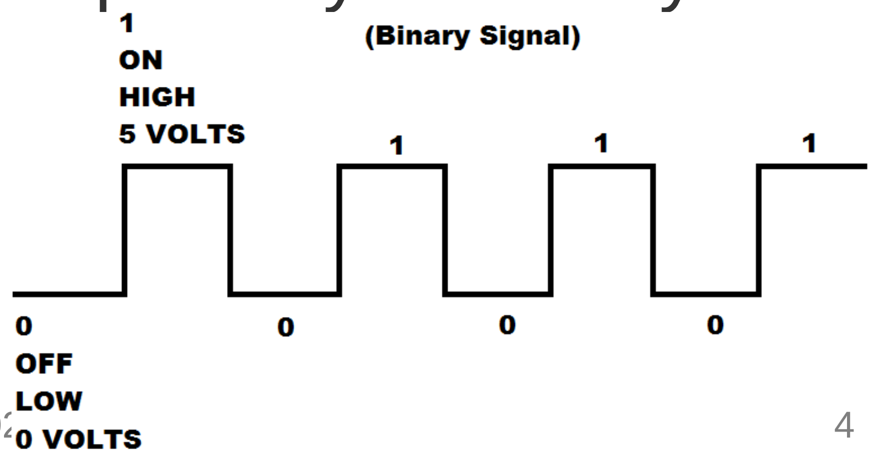
- Number Representation
  - Number Systems
  - Integers
    - Unsigned Integer
    - Signed Integer
  - Floating-Point Numbers
    - Unsigned Binary Fraction
    - Floating-Point Number Representation
- Character Representation
  - ASCII

Number Systems

- Common number systems:
  - The *radix* or *base* of the number system denotes the number of digits used in the system.

Binary (*base* 2)	**0 1**
Octal (*base* 8)	0 1 2 3 4 5 6 7
Decimal (*base* 10)	**0 1 2 3 4 5 6 7 8 9**
Hexadecimal (*base* 16)	0 1 2 3 4 5 6 7 8 9 A B C D E F

- The most natural way in a computer system is by binary numbers (0, 1).
  - (0, 1) can be represented as (off, on) electrical signals.



https://social.technet.microsoft.com/wiki/contents/articles/22118.declaring-numeric-data-types.aspx

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Count to 100 in Decimal!

$$\color{red}{1}\color{green}{0}\color{blue}{0} = \color{red}{1} \times 10^2 + \color{green}{0} \times 10^1 + \color{blue}{0} \times 10^0$$

https://www.youtube.com/watch?v=U6BCbH3Vwwk

“Unsigned” Integer Representation

- Consider an *n*-bit (or *n*-digit) vector

$$B = (b_{n-1} \dots b_1 b_0)_2,$$

Denoting the base as a **subscript**

where $b_i = 0 \; or \; 1$ (binary number) for $0 \leq i \leq n-1$

  - Most Significant Bit (MSB): b_{n-1} (i.e., the leftmost bit)
  - Least Significant Bit (LSB): b_0 (i.e., the rightmost bit)

- This vector can represent the decimal value for an unsigned integer $\mathrm{V}(B)$ in the range 0 to $2^n - 1$, where

$$\mathrm{V}(B) = b_{n-1} \times 2^{n-1} + \cdots + b_1 \times 2^1 + b_0 \times 2^0$$

- For example, if $B = (1001)_2, where \; n = 4$

$$\mathrm{V}(B) = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (9)_{10}$$

Conversion of Number Systems

(Decimal)$_{10}$	(Binary)$_2$	(Octal)$_8$	(Hexadecimal)$_{16}$
$(00)_{10}$	$(0000)_2$	$(00)_8$	$(0)_{16}$
$(01)_{10}$	$(0001)_2$	$(01)_8$	$(1)_{16}$
$(02)_{10}$	$(0010)_2$	$(02)_8$	$(2)_{16}$
$(03)_{10}$	$(0011)_2$	$(03)_8$	$(3)_{16}$
$(04)_{10}$	$(0100)_2$	$(04)_8$	$(4)_{16}$
$(05)_{10}$	$(0101)_2$	$(05)_8$	$(5)_{16}$
$(06)_{10}$	$(0110)_2$	$(06)_8$	$(6)_{16}$
$(07)_{10}$	$(0111)_2$	$(07)_8$	$(7)_{16}$
$(08)_{10}$	$(1000)_2$	$(10)_8$	$(8)_{16}$
$(09)_{10}$	$(1001)_2$	$(11)_8$	$(9)_{16}$
$(10)_{10}$	$(1010)_2$	$(12)_8$	$(A)_{16}$
$(11)_{10}$	$(1011)_2$	$(13)_8$	$(B)_{16}$
$(12)_{10}$	$(1100)_2$	$(14)_8$	$(C)_{16}$
$(13)_{10}$	$(1101)_2$	$(15)_8$	$(D)_{16}$
$(14)_{10}$	$(1110)_2$	$(16)_8$	$(E)_{16}$
$(15)_{10}$	$(1111)_2$	$(17)_8$	$(F)_{16}$

Every 4 bits → 1 hex digit

Class Excercise 2.1

Student ID: ____________ Date:
Name: __________________ ________

- Represent $(255)_{10}$ in binary, octal, and hexadecimal:

Binary (*base* 2)	0 1
Octal (*base* 8)	0 1 2 3 4 5 6 7
Decimal (*base* 10)	0 1 2 3 4 5 6 7 8 9
Hexadecimal (*base* 16)	0 1 2 3 4 5 6 7 8 9 A B C D E F

Addition of "Unsigned" Integers

- Addition of 1-bit unsigned numbers:

$$\begin{array}{r} 0 \\ +\ \ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ +\ \ 0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 0 \\ +\ \ 1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\ \ 1 \\ \hline 10 \end{array}$$

carry-out sum

- To add multiple-bit numbers:
  - We add bit pairs starting from the low-order (right) end, propagating carries toward the high-order (left) end.
    - The carry-out from a bit pair becomes the carry-in to the next bit pair.
    - The carry-in must be added to a bit pair in generating the sum and carry-out at that position.
    - For example,

carry-in 1

$$\begin{array}{rr} & 01111111 \\ + & 00000001 \\ \hline & \mathbf{10000000} \end{array}$$

high-order ← low-order

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“Signed” Integer Representation (1/3)

- To represent both positive and negative numbers, we need different systems to represent signed integer.

- In *written* decimal system, a signed integer is usually represented by a “+” or “–” sign and followed by the magnitude.
  - E.g. -73, -215, $+349$

- In binary system, we have three common systems:
  ① **Sign-and-magnitude**
  ② **1’s-complement**
  ③ **2’s-complement**

“Signed” Integer Representation (2/3)

- The leftmost bit (MSB) decides the sign (0: “+”, 1: “–”).

Positive values are identical in all the three systems:

- *Rule*: Treating the rest bits as an unsigned integer
  - E.g., +3 is represented by 0011.

Negative values have different representations:

① **Sign-and-magnitude** (MSB: sign, other bits: magnitude)

- *Rule*: Changing the MSB from 0 to 1
  - E.g. –3 is represented by 1011.

```
s&m: 0011
      ↓
     1011
```

② **1’s-complement**

- *Rule*: Inverting each bit of the positive number
  - E.g. –3 is obtained by flipping each bit in 0011 to yield 1100.

```
1’s: 0011
     ↓↓↓↓
     1100
```

③ **2’s-complement**

- *Rule 1*: Subtracting the positive number from the unsigned 2^n
- *Rule 2*: Adding 1 to 1’s-complement of that negative number
  - E.g. –3 is represented by 1101 when applying either rule.

```
2’s r1:
  10000
-)  0011
-------
   1101
2’s r2:
   1100
+) 0001
-------
   1101
```

“Signed” Integer Representation (3/3)

B	Values Represented in Decimal		
$b_3b_2b_1b_0$	Sign-and-magnitude	1’s-complement	2’s-complement
0 1 1 1	+ 7	+ 7	+ 7
0 1 1 0	+ 6	+ 6	+ 6
0 1 0 1	+ 5	+ 5	+ 5
0 1 0 0	+ 4	+ 4	+ 4
0 0 1 1	+ 3	+ 3	+ 3
0 0 1 0	+ 2	+ 2	+ 2
0 0 0 1	+ 1	+ 1	+ 1
0 0 0 0	+ 0	+ 0	+ 0
1 0 0 0	− 0	− 7	**− 8**
1 0 0 1	− 1	− 6	− 7
1 0 1 0	− 2	− 5	− 6
1 0 1 1	− 3	− 4	− 5
1 1 0 0	− 4	− 3	− 4
1 1 0 1	− 5	− 2	− 3
1 1 1 0	− 6	− 1	− 2
1 1 1 1	− 7	− 0	− 1

Class Exercise 2.2

- Question: Consider the decimal number `-56`. Please use 8 bits to represent it in:
  - Sign-and-magnitude: ____________________
  - 1's-complement: ____________________
  - 2's-complement: ____________________

- Question: Consider the 8-bit vector `10110101`, what is its decimal value when interpreted as:
  - Sign-and-magnitude: ____________________
  - 1's-complement: ____________________
  - 2's-complement: ____________________

Arithmetic of "Signed" Integers

- The three signed integer representation systems differ only in the way of representing negative values.

- Their relative merits on performing arithmetic operations can be summarized as follows:
  - **Sign-and-magnitude**: the simplest representation, but it is also the most awkward for addition/subtraction operations.
  - **1's-complement**: somewhat better than the sign-and-magnitude system.
  - **2's-complement**: specially designed to be efficient in performing addition and subtraction operations.
    - This is also why the 2's-complement system is the one most often used in modern computers.

Why 2's-complement Arithmetic?

- First consider adding $+7$ to -3:
  - What if we perform this addition by adding bit pairs from right to left (as what we did for n-bit unsigned numbers)?

$$
\begin{array}{rcccc}
 & 0 & 1 & 1 & 1 \\
+ & 1 & 1 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 0
\end{array}
$$

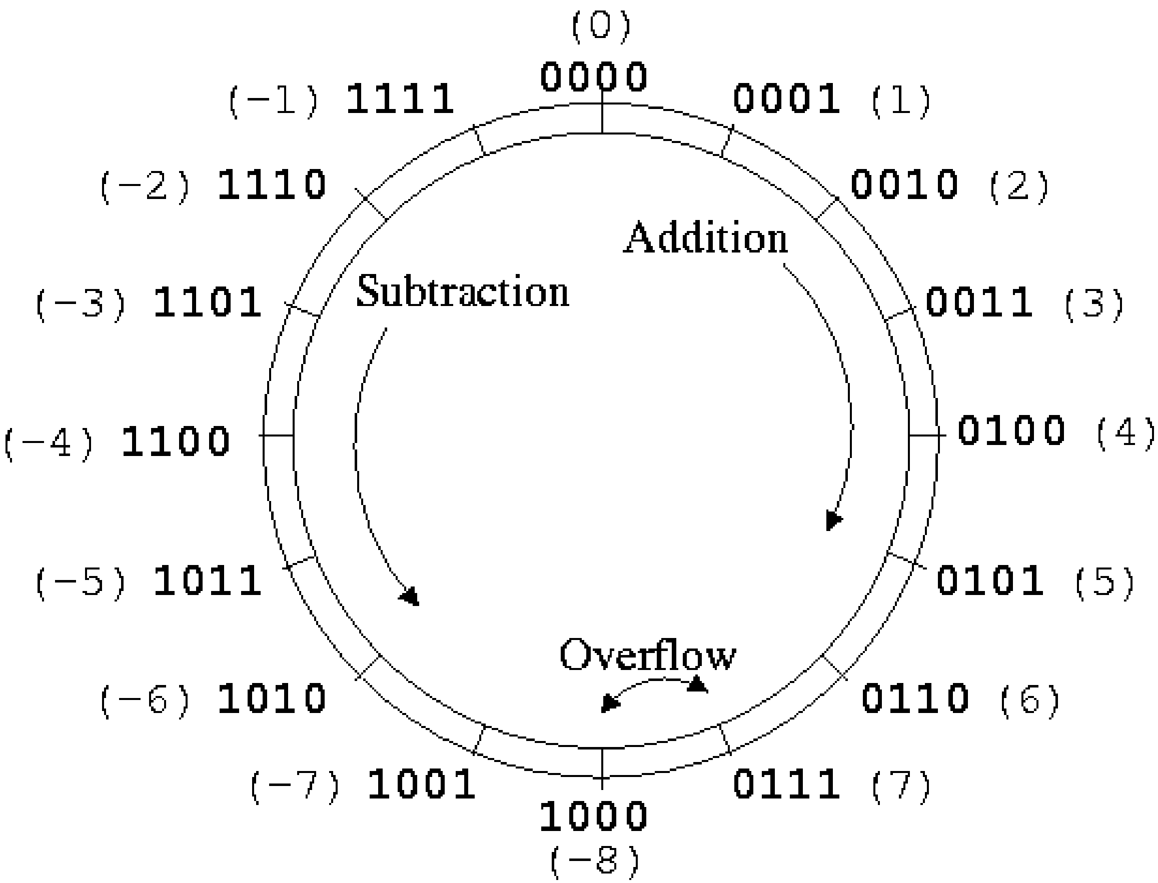
leftmost carry-out bit

  - If the leftmost carry-out bit is ignored, we get $(+4)_{10}$.
- Rules for *n*-bit signed number addition/subtraction:
  - $\boldsymbol{X+Y}$
    - Add their n-bit 2's-complement representations from right to left
    - Ignore the carry-out bit at the MSB position
  - $\boldsymbol{X-Y}$
    - Interpret as, and perform $X+(-Y)$
  - *Note: The sum should be in the range of* $-2^{n-1} \sim (2^{n-1}-1)$

Class Exercise 2.3

- Using 4-bit 2's-complement number to calculate:
  - `2+3`
  - `4+(-6)`
  - `(-5)+(-2)`
  - `2-4`
  - `(-7)-1`
  - `(-7)-(-5)`

2's-Complement Number Wheel



B	Decimal Value	
$b_3b_2b_1b_0$	1's-comp.	2's-comp
0 1 1 1	+ 7	+ 7
0 1 1 0	+ 6	+ 6
0 1 0 1	+ 5	+ 5
0 1 0 0	+ 4	+ 4
0 0 1 1	+ 3	+ 3
0 0 1 0	+ 2	+ 2
0 0 0 1	+ 1	+ 1
0 0 0 0	+ 0	+ 0
1 0 0 0	− 7	**− 8**
1 0 0 1	− 6	− 7
1 0 1 0	− 5	− 6
1 0 1 1	− 4	− 5
1 1 0 0	− 3	− 4
1 1 0 1	− 2	− 3
1 1 1 0	− 1	− 2
1 1 1 1	− ~~0~~	− 1

adding one

https://stackoverflow.com/questions/55145028/binary-ones-complement-in-python-3/55145758

Overflow in Integer Arithmetic

- **Overflow**: The result of an arithmetic operation does not fall within the representable range.
  - In **Unsigned** Number Arithmetic:
    - *Rule*: A carry-out of 1 from the MSB-bit always indicates an overflow.
      - E.g. $(1111)_2 + (0001)_2 = (\underline{1}\ 0000)_2$ ← *overflowed*
      - E.g. $(0111)_2 + (0001)_2 = (0\ 1000)_2$ ← *no overflow*
  - In **2's-complement Signed** Number Arithmetic:
    - The carry-out bit from the sign-bit is not an indicator of overflow.
      - E.g. $(+7)_{10} + (+4)_{10} = (0111)_2 + (0100)_2 = (\underline{0}\ 1011)_2 = (-5)_{10}$
      - E.g. $(-4)_{10} + (-6)_{10} = (1100)_2 + (1010)_2 = (\underline{1}\ 0110)_2 = (+6)_{10}$
    - *Observation*: Addition of opposite sign numbers *never* causes overflow.
      - E.g. $(+7)_{10} + (-6)_{10} = (0111)_2 + (1010)_2 = (0001)_2 = (+1)_{10}$ ← *no overflow*
    - *Rule*: If the two numbers are the same sign and the result is the opposite sign, we say that an overflow has occurred.
      - E.g. $(+7)_{10} + (+4)_{10} = (0111)_2 + (0100)_2 = (1011)_2 = (-5)_{10}$ ← *overflowed*
      - E.g. $(-4)_{10} + (-6)_{10} = (1100)_2 + (1010)_2 = (0110)_2 = (+6)_{10}$ ← *overflowed*

Sign Extension

- We often need to represent a value given in a certain number of bits by using a larger number of bits.
  - That is, how to represent a signed integer by using a larger number of bits?
- Sign Extension: Simply repeat the "sign bit" as many digits as needed to the left. (*Note: It can be applied to both 1's and 2's-complement, but not sign-and-magnitude*)
  - Positive Number: Add 0's to the **left-hand-side**
    - E.g. 0111 → 0000 0111
  - Negative Number: Add 1's to the **left-hand-side**
    - E.g. 1010 → 1111 1010

Example: Representing −2~+1 with 8 bits by 2's-complement

B = $b_7b_6…b_0$	2's complement
000000 01	+1
000000 00	+0
111111 10	-2
111111 11	-1

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Unsigned Binary Fraction

- Consider a *n*-bit unsigned binary fraction:

$$B = (0.b_{-1}b_{-2} \dots b_{-n})_2$$

where $b_{-i} = 0 \; or \; 1$ (binary number) for $1 \leq i \leq n$

- This vector can represent the value for an unsigned binary fraction $\mathrm{F}(B)$, where

$$\mathrm{F}(B) = b_{-1} \times 2^{-1} + b_{-2} \times 2^{-2} + \cdots + b_{-n} \times 2^{-n}$$

- The range of $\mathrm{F}(B)$ is

Why? Geometric Series

$$S_n = \sum_{i=1}^{n} a_i r^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$0 \leq \mathrm{F}(B) \leq \underline{1 - 2^{-n}}$$

$$0 \leq \mathrm{F}(B) \approx +1.0, \textit{ for a large } n$$

Binary Fraction to Decimal Fraction

- What is the binary fraction $(0.01101)_2$ in decimal ?
- Method 1:

$$(0.01101)_2 = 2^{-2} + 2^{-3} + 2^{-5} = (0.40625)_{10}$$

- Method 2:

.	0	1	1	0	1
32	16	8	4	2	1

$8+4+1=13$

$$13/_{32} = (0.40625)_{10}$$

Decimal Fraction to Binary Fraction

- What is the decimal fraction $(0.6875)_{10}$ in binary ?

```
0.6875 * 2 = 1.3750  →  0.1???₂
0.3750 * 2 = 0.7500  →  0.10??₂
0.7500 * 2 = 1.5000  →  0.101?₂
0.5000 * 2 = 1.0000  →  0.1011₂
0.0000 * 2 = 0       →  End
```

- Answer: $\mathbf{(0.1011)_2}$

Why? Let's have an analogy in decimal:

```
0.6875 * 10 = 6.875   →  (0.6???)₁₀
0.8750 * 10 = 8.7500  →  (0.68??)₁₀
...
```

Class Exercise 2.4

- What is the decimal fraction $(0.1)_{10}$ in binary ?
- Answer:

What did we learn so far?

- On one hand :
  - Some decimal fractions (e.g., $(0.1)_{10}$) will produce infinite binary fraction expansions.
  - A n-bit unsigned fraction can only represent values in the range of $0 \sim 1-2^{-n}$ and cannot represent negative values.
  - The position of the binary point in a floating-point number varies (that's way called floating point!).

    `0.232 * 10`^{4} `= 2.320000 * 10`^{3} `= 23.20000 * 10`^{2} `= …`
- On the other hand:
  - A n-bit signed integer in 2's-complement form can only represent values in the range of $-2^{(n-1)} \sim 2^{(n-1)}-1$.
- We need a unique representation (form) that can
  ① Represent the sign, and the position of the floating point.
  ② Represent very large integers and very small fractions.

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Floating Point Number Representation

- In decimal scientific notation, numbers are written as :
  $+6.0247 \times 10^{23}$, $+3.7291 \times 10^{-27}$, -7.3000×10^{-14}, ...

- The same approach can be used to represent binary floating-point numbers (using 2 as the base) by:
  - Sign: A sign for the number
  - Mantissa: Some significant bits
  - Exponent: A signed scale factor (implied base of 2)

- To have a normalized representation for floating-point numbers, we should normalize Mantissa in the range $[1 \ldots B)$, where B is the base.
  - Binary System: $[1 \ldots 2)$
    - $(1.b_{-1}b_{-2}\ldots b_{-n})_2$ must in the range of $[1 \ldots 2)$.

IEEE Standard 754 Single Precision

- The **single** precision format is a 32-bit representation.
  - The leftmost bit represents the sign, S, for the number.
  - The next 8 bits, E', represent the **unsigned integer** for the $excess - 127$ *exponent* (with base of 2).
    - **Note:** The **actual signed exponent E** is E'–127
  - The remaining 23 bits, M, are the significant bits.

32 bits

S	E'	M

Sign of number : 0 signifies + 1 signifies -

$\underbrace{\quad E' \quad}$ 8-bit excess-127 exponent *range of E': 0~255*

$\underbrace{\quad M \quad}$ 23-bit mantissa fraction

Note: No need to represent the leading 1 in *M*.

Note: The value of ***M*** is treated as unsigned binary fraction.

Value represented $= \pm \mathbf{1}.M \times 2^{E' - 127}$

Example:

0	0 0 1 0 1 0 0 0	0 1 1 0 1 0 . . . 0

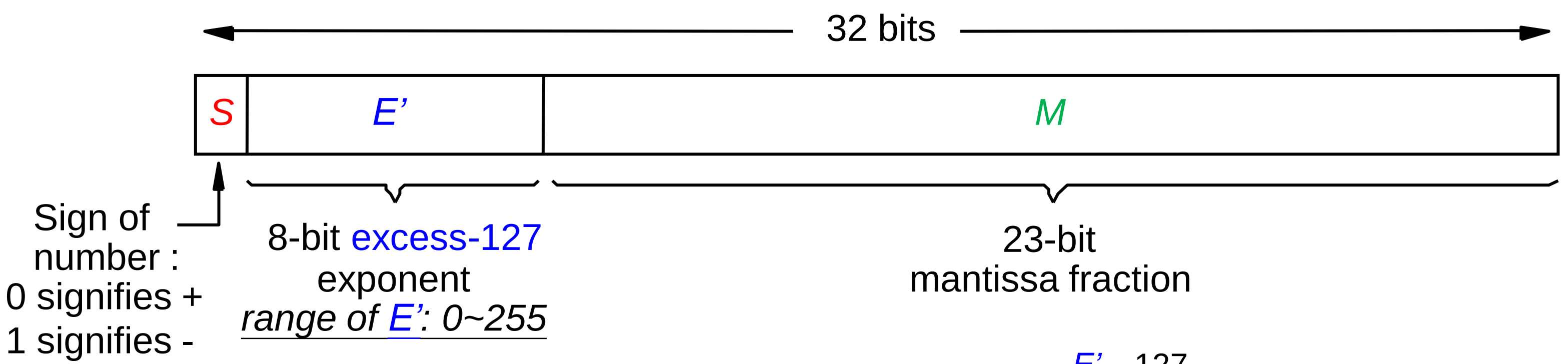
$(00101000)_2 \rightarrow (40)_{10}$
$40 - 127 = \mathbf{-87}$

Value represented $= +\mathbf{1}.\underline{01101\ldots0} \times 2^{-87}$

$(0.01101\ldots0)_2 \rightarrow (0.40625)_{10}$

Class Exercise 2.5

- What is the IEEE single precision number $(\texttt{40C0 0000})_{16}$ in decimal?



Value represented $= \pm 1.M \times 2^{E' - 127}$

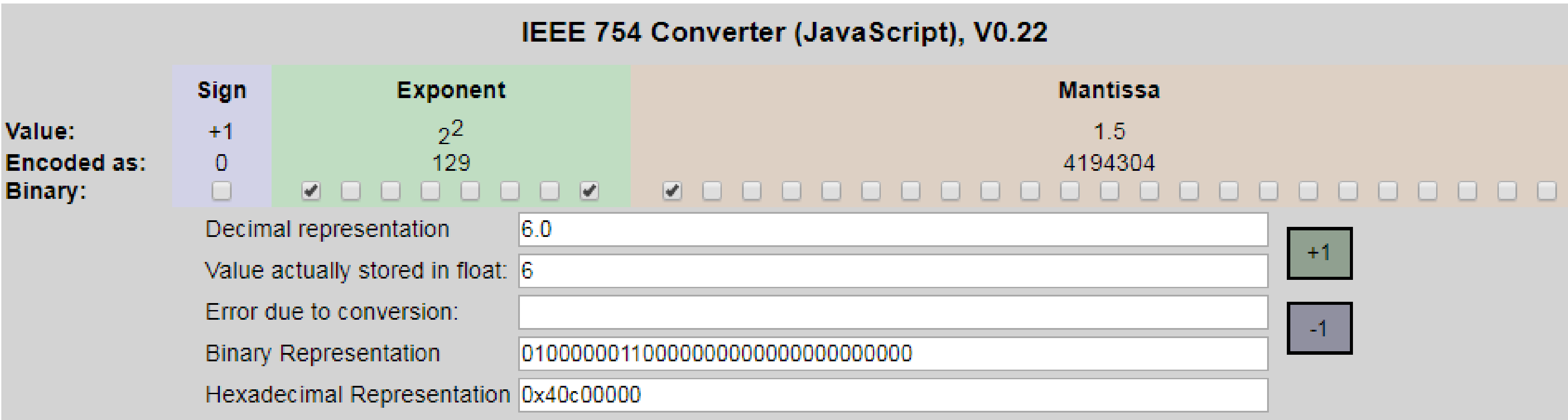
- Answer:

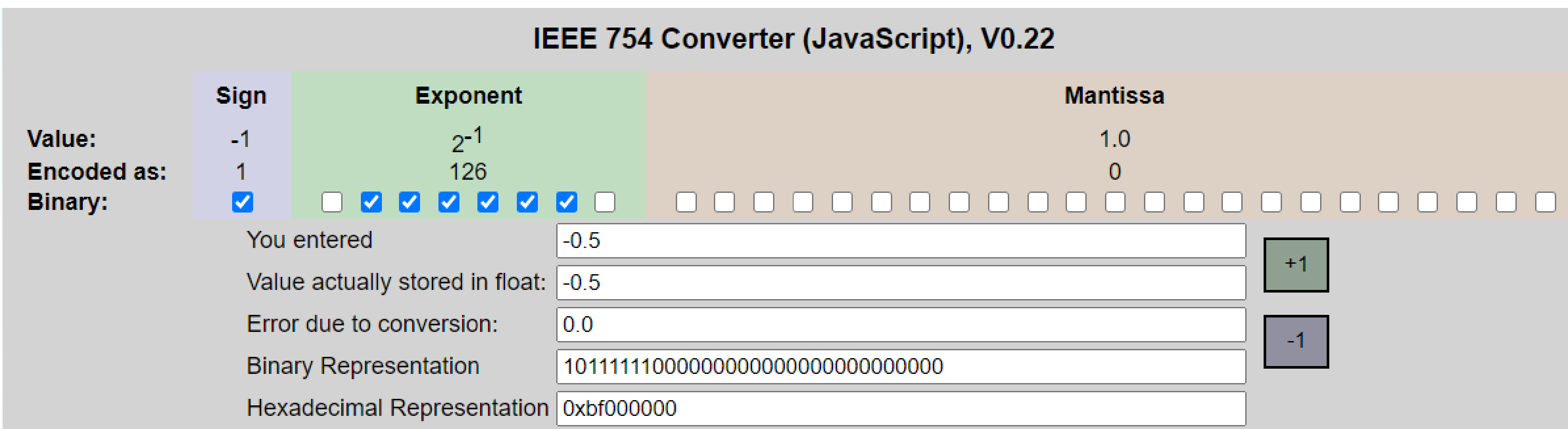
Class Exercise 2.6

- What is $(-0.5)_{10}$ in the IEEE single precision binary floating point format?
- Answer:

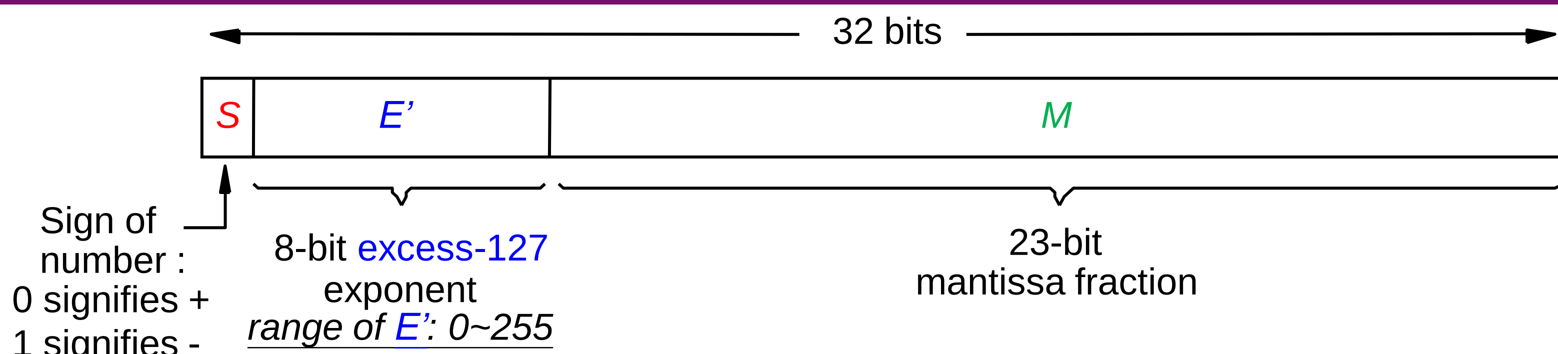
Useful Tool

- IEEE-754 Floating Point Converter
  - https://www.h-schmidt.net/FloatConverter/IEEE754.html





Special Values



Value represented = $\pm 1.M \times 2^{E' - 127}$

- When exponent $E' = 0$ (all 0's) and mantissa $M = 0$:
  - The value 0 is represented.
- When exponent $E' = 0$ (all 0's) and mantissa $M \neq 0$:
  - $Denormal\ values$ (i.e. very small values) are represented.
- When exponent $E' = 255$ (all 1's) and mantissa $M = 0$:
  - The value ∞ is presented.
- When exponent $E' = 255$ (all 1's) and mantissa $M \neq 0$:
  - $Not\ a\ Number\ (NaN)$ (e.g. $0/0$ or $\sqrt{-1}$) is presented.
- Check this article for more information.

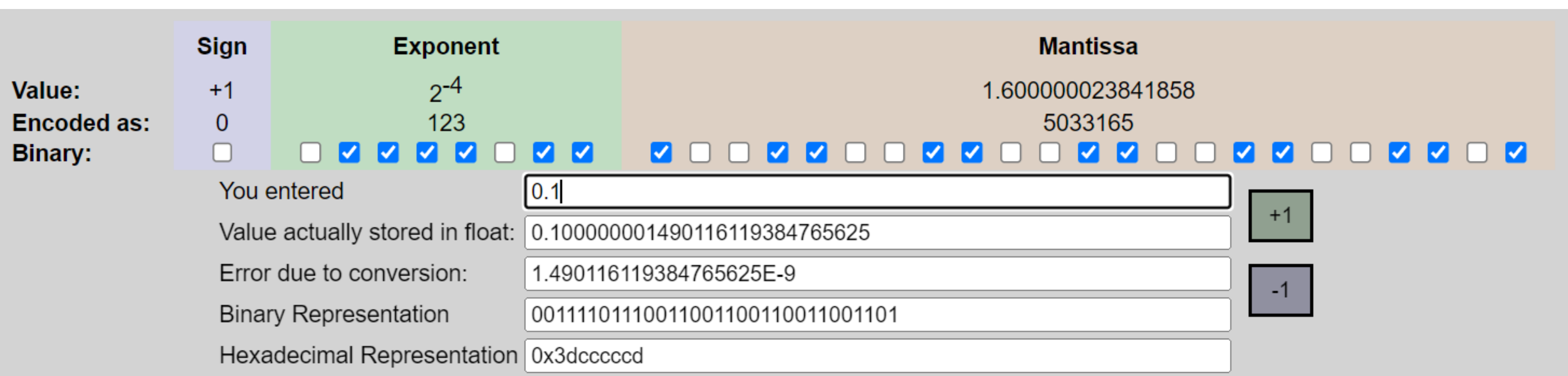
Revisit: $(0.1)_{10}$ in Binary (1/2)

- Unsigned Binary Fraction (32-bit)

$$(0.0\ 0011\ 0011\ 0011\ 0011\ 0011\ 0011\ 0011\ 001)_2$$

- IEEE-754 Single Precision (32-bit)

$$0\ 01111011\ 1001\ 1001\ 1001\ 1001\ 1001\ 101$$
$$= +(1.1001\ 1001\ 1001\ 1001\ 1001\ 101)_2 \times 2^{-4}$$
$$= (0.000\ 11001\ 1001\ 1001\ 1001\ 1001\ 101)_2$$



Revisit: $(0.1)_{10}$ in Binary (2/2)

- Unsigned Binary Fraction
- IEEE Single Precision

From: Binary | To: Decimal

Enter binary number

0.00011001100110011001100110011001 (base 2)

= Convert | × Reset | ⇄ Swap

Decimal number

0.09999999986030161381 (base 10)

Decimal from signed 2's complement

N/A (base 10)

Hex number

0.19999999000000000AF7 (base 10)

Decimal calculation steps

$(0.00011001100110011001100110011001)_2 = (0 \times 2^{0}) + (0 \times 2^{-1}) + (0 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) + (1 \times 2^{-5}) + (0 \times 2^{-6}) + (0 \times 2^{-7}) + (1 \times 2^{-8}) + (1 \times 2^{-9}) + (0 \times 2^{-10}) + (0 \times 2^{-11}) + (1 \times 2^{-12}) + (1 \times 2^{-13}) + (0 \times 2^{-14}) + (0 \times 2^{-15}) + (1 \times 2^{-16}) + (1 \times 2^{-17}) + (0 \times 2^{-18}) + (0 \times 2^{-19}) + (1 \times 2^{-20}) + (1 \times 2^{-21}) + (0 \times 2^{-22}) + (0 \times 2^{-23}) + (1 \times 2^{-24}) + (1 \times 2^{-25}) + (0 \times 2^{-26}) + (0 \times 2^{-27}) + (1 \times 2^{-28}) + (1 \times 2^{-29}) + (0 \times 2^{-30}) + (0 \times 2^{-31}) + (1 \times 2^{-32}) = (0.09999999986030161381)_{10}$

```
   0.1
-) 0.09999999986030161381
-------------------------
   0.00000000013969838619
```

```
   0.10000000149011611938
-) 0.1
-------------------------
   0.00000000149011611938
```

From: Binary | To: Decimal

Enter binary number

0.0001100110011001100110011001101 (base 2)

= Convert | × Reset | ⇄ Swap

Decimal number

0.10000000149011611938 (base 10)

Decimal from signed 2's complement

N/A (base 10)

Hex number

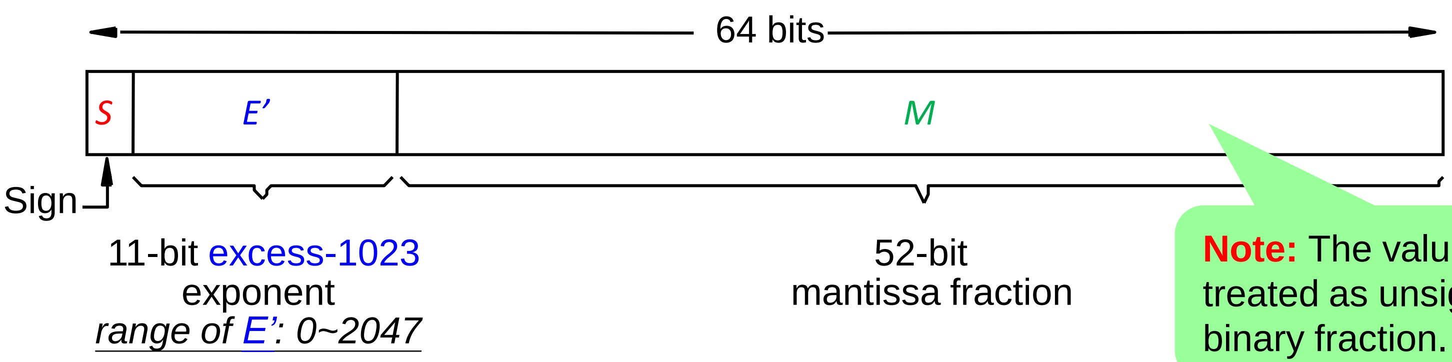
0.1999999FFFFFFFFFE97F (base 10)

Decimal calculation steps

$(0.0001100110011001100110011001101)_2 = (0 \times 2^{0}) + (0 \times 2^{-1}) + (0 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) + (1 \times 2^{-5}) + (0 \times 2^{-6}) + (0 \times 2^{-7}) + (1 \times 2^{-8}) + (1 \times 2^{-9}) + (0 \times 2^{-10}) + (0 \times 2^{-11}) + (1 \times 2^{-12}) + (1 \times 2^{-13}) + (0 \times 2^{-14}) + (0 \times 2^{-15}) + (1 \times 2^{-16}) + (1 \times 2^{-17}) + (0 \times 2^{-18}) + (0 \times 2^{-19}) + (1 \times 2^{-20}) + (1 \times 2^{-21}) + (0 \times 2^{-22}) + (0 \times 2^{-23}) + (1 \times 2^{-24}) + (1 \times 2^{-25}) + (0 \times 2^{-26}) + (1 \times 2^{-27}) = (0.10000000149011611938)_{10}$

IEEE Standard 754 Double Precision

- The **double** precision format is a 64-bit representation.
  - The leftmost bit represents the sign, S, for the number.
  - The next 11 bits, E’, represent the **unsigned integer** for the *excess-1023 exponent* (with base of 2).
    - **Note:** The actual **signed** exponent E is E’–1023.
  - The remaining 52 bits, M, are the significant bits.



Note: The value of *M* is treated as unsigned binary fraction.

Value represented = $\pm 1.M \times 2^{E' - 1023}$

Note: No need to represent the leading 1 in *M*.

Outline

- Number Representation
  - Number Systems
  - Integers
    - Unsigned Integer
    - Signed Integer
  - Floating-Point Numbers
    - Unsigned Binary Fraction
    - Floating-Point Number Representation

- Character Representation
  - ASCII

Character Representation

- The most common encoding scheme for **characters** is ASCII (American Standard Code for Information Interchange).

- In ASCII encoding scheme, alphanumeric characters, operators, punctuation symbols, and control characters can be represented by 7-bit codes.
  - It is convenient to use an 8-bit *byte* to represent a character.
    - The code occupies the low-order 7 bits with the high-order bit as 0.

- Extended ASCII encoding scheme uses 8-bit (or even more) to represent the standard 7-bit ASCII characters, plus additional characters.

ASCII Table

Dec	Bin	Hex	Char	Dec	Bin	Hex	Char	Dec	Bin	Hex	Char	Dec	Bin	Hex	Char
0	0000 0000	00	[NUL]	32	0010 0000	20	space	64	0100 0000	40	@	96	0110 0000	60	`
1	0000 0001	01	[SOH]	33	0010 0001	21	!	65	0100 0001	41	A	97	0110 0001	61	a
2	0000 0010	02	[STX]	34	0010 0010	22	"	66	0100 0010	42	B	98	0110 0010	62	b
3	0000 0011	03	[ETX]	35	0010 0011	23	#	67	0100 0011	43	C	99	0110 0011	63	c
4	0000 0100	04	[EOT]	36	0010 0100	24	$	68	0100 0100	44	D	100	0110 0100	64	d
5	0000 0101	05	[ENQ]	37	0010 0101	25	%	69	0100 0101	45	E	101	0110 0101	65	e
6	0000 0110	06	[ACK]	38	0010 0110	26	&	70	0100 0110	46	F	102	0110 0110	66	f
7	0000 0111	07	[BEL]	39	0010 0111	27	'	71	0100 0111	47	G	103	0110 0111	67	g
8	0000 1000	08	[BS]	40	0010 1000	28	(	72	0100 1000	48	H	104	0110 1000	68	h
9	0000 1001	09	[TAB]	41	0010 1001	29	)	73	0100 1001	49	I	105	0110 1001	69	i
10	0000 1010	0A	[LF]	42	0010 1010	2A	*	74	0100 1010	4A	J	106	0110 1010	6A	j
11	0000 1011	0B	[VT]	43	0010 1011	2B	+	75	0100 1011	4B	K	107	0110 1011	6B	k
12	0000 1100	0C	[FF]	44	0010 1100	2C	,	76	0100 1100	4C	L	108	0110 1100	6C	l
13	0000 1101	0D	[CR]	45	0010 1101	2D	-	77	0100 1101	4D	M	109	0110 1101	6D	m
14	0000 1110	0E	[SO]	46	0010 1110	2E	.	78	0100 1110	4E	N	110	0110 1110	6E	n
15	0000 1111	0F	[SI]	47	0010 1111	2F	/	79	0100 1111	4F	O	111	0110 1111	6F	o
16	0001 0000	10	[DLE]	48	0011 0000	30	0	80	0101 0000	50	P	112	0111 0000	70	p
17	0001 0001	11	[DC1]	49	0011 0001	31	1	81	0101 0001	51	Q	113	0111 0001	71	q
18	0001 0010	12	[DC2]	50	0011 0010	32	2	82	0101 0010	52	R	114	0111 0010	72	r
19	0001 0011	13	[DC3]	51	0011 0011	33	3	83	0101 0011	53	S	115	0111 0011	73	s
20	0001 0100	14	[DC4]	52	0011 0100	34	4	84	0101 0100	54	T	116	0111 0100	74	t
21	0001 0101	15	[NAK]	53	0011 0101	35	5	85	0101 0101	55	U	117	0111 0101	75	u
22	0001 0110	16	[SYN]	54	0011 0110	36	6	86	0101 0110	56	V	118	0111 0110	76	v
23	0001 0111	17	[ETB]	55	0011 0111	37	7	87	0101 0111	57	W	119	0111 0111	77	w
24	0001 1000	18	[CAN]	56	0011 1000	38	8	88	0101 1000	58	X	120	0111 1000	78	x
25	0001 1001	19	[EM]	57	0011 1001	39	9	89	0101 1001	59	Y	121	0111 1001	79	y
26	0001 1010	1A	[SUB]	58	0011 1010	3A	:	90	0101 1010	5A	Z	122	0111 1010	7A	z
27	0001 1011	1B	[ESC]	59	0011 1011	3B	;	91	0101 1011	5B	[	123	0111 1011	7B	{
28	0001 1100	1C	[FS]	60	0011 1100	3C	<	92	0101 1100	5C	\	124	0111 1100	7C	|
29	0001 1101	1D	[GS]	61	0011 1101	3D	=	93	0101 1101	5D	]	125	0111 1101	7D	}
30	0001 1110	1E	[RS]	62	0011 1110	3E	>	94	0101 1110	5E	^	126	0111 1110	7E	~
31	0001 1111	1F	[US]	63	0011 1111	3F	?	95	0101 1111	5F	_	127	0111 1111	7F	[DEL]

Extended ASCII Table

ASCII control characters		
00	NULL	(Null character)
01	SOH	(Start of Header)
02	STX	(Start of Text)
03	ETX	(End of Text)
04	EOT	(End of Trans.)
05	ENQ	(Enquiry)
06	ACK	(Acknowledgement)
07	BEL	(Bell)
08	BS	(Backspace)
09	HT	(Horizontal Tab)
10	LF	(Line feed)
11	VT	(Vertical Tab)
12	FF	(Form feed)
13	CR	(Carriage return)
14	SO	(Shift Out)
15	SI	(Shift In)
16	DLE	(Data link escape)
17	DC1	(Device control 1)
18	DC2	(Device control 2)
19	DC3	(Device control 3)
20	DC4	(Device control 4)
21	NAK	(Negative acknowledge)
22	SYN	(Synchronous idle)
23	ETB	(End of trans. block)
24	CAN	(Cancel)
25	EM	(End of medium)
26	SUB	(Substitute)
27	ESC	(Escape)
28	FS	(File separator)
29	GS	(Group separator)
30	RS	(Record separator)
31	US	(Unit separator)
127	DEL	(Delete)

ASCII printable characters					
32	space	64	@	96	`
33	!	65	A	97	a
34	"	66	B	98	b
35	#	67	C	99	c
36	$	68	D	100	d
37	%	69	E	101	e
38	&	70	F	102	f
39	'	71	G	103	g
40	(	72	H	104	h
41	)	73	I	105	i
42	*	74	J	106	j
43	+	75	K	107	k
44	,	76	L	108	l
45	-	77	M	109	m
46	.	78	N	110	n
47	/	79	O	111	o
48	0	80	P	112	p
49	1	81	Q	113	q
50	2	82	R	114	r
51	3	83	S	115	s
52	4	84	T	116	t
53	5	85	U	117	u
54	6	86	V	118	v
55	7	87	W	119	w
56	8	88	X	120	x
57	9	89	Y	121	y
58	:	90	Z	122	z
59	;	91	[	123	{
60	<	92	\	124	|
61	=	93	]	125	}
62	>	94	^	126	~
63	?	95	_		

Extended ASCII characters							
128	Ç	160	á	192	└	224	Ó
129	ü	161	í	193	┴	225	ß
130	é	162	ó	194	┬	226	Ô
131	â	163	ú	195	├	227	Ò
132	ä	164	ñ	196	─	228	õ
133	à	165	Ñ	197	┼	229	Õ
134	å	166	ª	198	ã	230	µ
135	ç	167	º	199	Ã	231	þ
136	ê	168	¿	200	╚	232	Þ
137	ë	169	®	201	╔	233	Ú
138	è	170	¬	202	╩	234	Û
139	ï	171	½	203	╦	235	Ù
140	î	172	¼	204	╠	236	ý
141	ì	173	¡	205	═	237	Ý
142	Ä	174	«	206	╬	238	¯
143	Å	175	»	207	¤	239	´
144	É	176	░	208	ð	240	≡
145	æ	177	▒	209	Ð	241	±
146	Æ	178	▓	210	Ê	242	‗
147	ô	179	│	211	Ë	243	¾
148	ö	180	┤	212	È	244	¶
149	ò	181	Á	213	ı	245	§
150	û	182	Â	214	Í	246	÷
151	ù	183	À	215	Î	247	¸
152	ÿ	184	©	216	Ï	248	°
153	Ö	185	╣	217	┘	249	¨
154	Ü	186	║	218	┌	250	·
155	ø	187	╗	219	█	251	¹
156	£	188	╝	220	▄	252	³
157	Ø	189	¢	221	¦	253	²
158	×	190	¥	222	Ì	254	■
159	ƒ	191	┐	223	▀	255	nbsp

Class Exercise 2.7

- Represent "`Hello, CSCI2510`" using ASCII code:

	Decimal	Binary
`H`		
`e`		
`l`		
`l`		
`o`		
`,`		
`C`		
`S`		
`C`		
`I`		
`2`		
`5`		
`1`		
`0`		

Summary

- Number Representation
  - Number Systems
  - Integers
    - Unsigned Integer
    - Signed Integer
  - Floating-Point Numbers
    - Unsigned Binary Fraction
    - Floating-Point Number Representation

- Character Representation
  - ASCII